

# Effects of Aggregate Shocks in Inventory Models: Shedding light about the crisis.<sup>1</sup>

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## Abstract

This paper studies the transmission channels of financial stresses to the real economy. Using an inventory-menu cost model, we study the dynamics of inventory distribution in response to main propagation channels: variable and fixed cost-push stresses and demand contraction. We propose an identification strategy through sign restrictions on the changes of the first and second order moments and the right tail of the inventory distribution, considering both short and long term responses. We examine firm-level data of the manufacturing and trade sectors of Peru and U.S. during the last financial crisis (2007-2009). We conclude that the main transmission mechanism for both sectors in U.S. was a variable cost-push stress, while in Peru transmission channels were different among sectors: a fixed cost-push stress for the trade sector and a demand contraction for the manufacturing sector.

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# 1 Introduction

The last financial crisis (2007-2009) was a stark reminder of how perverse effects financial stresses can produce on real economy. There is consensus on the direction of propagation, from the financial sector to the real one, and on the presence of several transmission mechanisms. However, the relative importance of those propagations channels is an issue that has not been successfully addressed. To what extent financial stresses propagate through demand contraction, and to what extent they do through cost-push stresses? This paper builds on the idea that the high sensibility of inventories might be able to answer this question.

In order to explore this possibility we develop an inventory model. Literature on inventories has been mainly motivated by the importance of inventory investment in business cycle which has been fairly documented in Blinder (1981) and Carpenter, Fazzari and Petersen (1994). Hence, it has mainly focused on inventories aggregate behavior: countercyclical pattern of inventory to sales ratios and strongly procyclical inventory investment and its mentioned importance in the business cycle. We depart from this venue and focus on the interaction between price, production and inventory decisions following the work of Aguirregabiria (1999) and Kryvtsov and Midrigan (2010). The model describes the optimal behavior of a risk-neutral firm with market power, which decides its price and production in a demand uncertainty framework. Each time the firm produces, it faces lump sum and variable costs, and each time it decides to change its price, it incurs in menu costs. The firm has the possibility of holding inventories facing storage costs and depreciation. Inventory holdings arise as an optimal response to demand uncertainty and the presence of lump sum costs. Results also reveal a standard  $(S, s)$  rule for production, and a negative substitution relationship between price and production decisions.

We evaluate the dynamics of inventory distribution in response to main propagation channels: variable and fixed cost-push stresses and demand contraction. Simulation exercises reveal that short term and long-term dynamics differ depending on the channel of transmission of the underlying shock. Particularly, when facing a lump sum cost-push stress, the firm increases its order size (or production level depending on the interpretation) and reduce its sales (through higher prices) in order to place orders more infrequently. This implies, on average, higher inventory holdings, and a larger cross section variance for production. Meanwhile, in the case of a variable cost-push stress, the firm increases prices and reduces order size to avoid paying disproportional storage costs. This reflects into lower inventory stocks and higher cross section production variance. Finally, in the case of a demand contraction, inventory holdings increase in the short term as consequence of lower than expected sales, but the optimal response of the firm leads to reduce inventory stocks in the long-term.

The detail analysis of the responses allows us to propose an identification strategy through sign restric-

tions on the changes of the first and second order moments and the right tail of the inventory distribution, considering both short and long term responses. We use this strategy to identify the main transmission channels of last financial crisis. We study plant-level data of manufacturing and trade sector of Peru and U.S. We find that the main transmission mechanism for both sectors in U.S. was a variable cost-push stress, while in Peru transmission channels were different among sectors: a fixed cost-push stress for the trade sector and a demand contraction for the manufacturing sector.

The rest of the paper proceeds as follows. Section 2 describes the inventory model. Section 3 presents results about the optimal decision rules, some workings of the model and a comparison of the effects generated by different aggregate shocks. Then, to understand what happened behind the great movements in data, Section 4 shows an approximation to the empirical data of manufacturing and trade sectors for Peru and U.S using the model proposed. Finally, Section 5 concludes.

## 2 Model

In this section we develop a model based on Aguirregabiria (1999) and Kryvtsov and Midrigan (2010). This is a partial equilibrium model that describes the inventory, production and price optimal decisions of a firm which faces an elastic demand. Holding inventories arise as an optimal response to demand fluctuations and to lump-sum ordering or production costs. The model delivers an  $(S, s)$  rule for inventory decision: whenever the inventory level falls below  $s$ , the firm will order as much as it needs to reach an inventory level of  $S$ . Besides, it reveals a negative correlation between price and available stock associated to the substitution relationship between these variables in the expected sales equation.

### 2.1 Environment

Consider a firm that sells an homogenous good which faces an elastic demand. We could assume that the firm produces this good or that it is a retailer that buys it from a wholesaler.<sup>2</sup> Every period  $t$ , the firm decides the price ( $p_t$ ) and the quantity order ( $q_t$ ) to maximize the expected and discounted stream of current and future profits. The firm makes its decisions prior to learning the value of the demand shock  $v_t$ . This assumption introduces the precautionary motive for holding inventories, the stock-out avoidance motive. Each period  $t$ , the expected profits are given by

$$E(\pi_t) = p_t \cdot E(y_t) - \alpha \cdot I_t - c(q_t) - F^q \cdot D(q_t > 0) - F^p \cdot D(p_t \neq p_{t-1}) \quad (1)$$

where  $E(y_t)$  are the expected sales,  $\alpha$  is the storage cost per unit,  $I_t$  is the inventory stock at the beginning of period  $t$  prior to the order decision,  $c(q_t)$  is the cost of an order of  $q_t$  units,  $F^q$  is the lump-sum cost of an order,  $F^p$  is the menu cost and  $D(\cdot)$  is a dummy variable which takes the value of one if the condition in brackets is satisfied and zero otherwise.

The quantity effectively sold is the minimum between inventories available for sale -the inventories at the beginning plus the period order- and the demand:

$$y_t = \min\{I_t + q_t, d(p_t, \tilde{p}_t, v_t)\}$$

where  $d(p_t, \tilde{p}_t, v_t)$  is the demand function which depends on the firm's price ( $p_t$ ), the demand shock ( $v_t$ ) and an index of every firm's price ( $\tilde{p}_t$ )<sup>3</sup>. Notice that this expression consider that there are no lags between the decision of ordering goods and the moment when these are add to the available stock. We will assume that the firm is small relative to the market where it operates, so it neglects the impact of its price on the

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<sup>2</sup>From here we will use indifferently production and orders.

<sup>3</sup>As we do not analyze the equilibrium, we assume the price index follows an stochastic exogenous process.

price index. We will also consider that the logarithm of demand shock follows a first order autoregressive process with a probability density function given by  $f_{v|v}(\cdot)$  and an isoelastic demand function, so expected sales given the previous values of the shock could be written as:

$$E(y_t|v_{t-1}) = \int \min\{I_t + q_t, \exp(v) \cdot A \cdot \left(\frac{p_t}{\bar{p}_t}\right)^{-\gamma}\} f_{v|v_{t-1}}(v) dv \quad (2)$$

where  $A$  is an scale parameter that could be interpreted as the market demand and  $\gamma$  is the price demand elasticity.

There exist the possibility of storing goods that are not sold, but there are costs associate with it. We allow for two type of storage costs. First, the firm has to pay  $\alpha$  for each unit it wants to storage until the next period, and second, there is a the depreciation rate  $(1 - R)$  that affects inventories, where  $R$  is the return on a marginal unit of inventory and it is lees than unity<sup>4</sup>. This implies that the inventories at the start of the next period are given by the following transition equation:

$$I_{t+1} = R(I_t + q_t - y_t) \quad (3)$$

Since the time horizon of the problem is infinite and considering a discount factor ( $\beta$ ) the firm's optimization problem in period  $t$ , given the demand shock of the previous period, the stock of inventories, and the firm's previous period price, can be written as <sup>5</sup>:

$$\max_{\{p_t, q_t \geq 0\}} \sum_{k=t}^{\infty} \beta^k \int \dots \int \left[ \int \pi_k dF_{v_k|v_{k-1}}(v_k) \right] dF_{v_{k-1}|v_{k-2}}(v_{k-1}) \dots dF_{v_t|v_{t-1}}(v_{t-1})$$

subject to the transition equation. The nonnegativity restriction for orders  $q_t \geq 0$  is, in fact, an irreversibility restriction: it prevents the firm from transforming its inventory into cash (at a price equal to production cost) in order to avoid storage costs. Note that the multiple integrals arise from allowing a dependence of the demand shock on the past realization of it.

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<sup>4</sup>This is to ensure that inventories will not multiply

<sup>5</sup>For simplicity, in the following expresion, we assume that the price index is constant and known by the firm. Otherwise, there should be several more integrals reflecting the uncertainty among the future price index. For example, if we assume it follows an iid process, we would have:

$$\max_{\{p_t, q_t \geq 0\}} \sum_{k=t}^{\infty} \beta^k \int_{\bar{p}} \int \dots \int \left[ \int \pi_k dF_{v_k|v_{k-1}}(v_k) \right] dF_{v_{k-1}|v_{k-2}}(v_{k-1}) \dots dF_{v_t|v_{t-1}}(v_{t-1}) f_{\bar{p}}(\tilde{p}) d\tilde{p}$$

## 2.2 Recursive Formulation

We next recast the problem recursively, at the beginning of period  $t$ , before the realization of the demand shock  $v_t$ , the state of the firm is characterized by its price in the preceding period ( $p_{t-1}$ ), its inventory stock at the beginning of the period ( $I_t$ ) and the previous realization of the demand shock ( $v_{t-1}$ )<sup>6</sup>. The control variables for the firm are its price  $p_t$  and the quantity of its order  $q_t$  while the transition equation is given by equation (3).

The presence of lump-sum costs makes the firm's decision become a dynamic stochastic discrete choice problem where the discrete choices are whether to produce or not and whether to change prices or not. With this, we can think the firm's problem in two steps: the first one where it maximizes the present value of its profits for each one of the four alternatives, and the second one where it chooses the alternative that provides the largest present value of the discounted profits. In brief, the firm's maximization problem can be written recursively as:

$$V(I, v, p) = \max \{V^i(I, v, p), V^p(I, v, p), V^q(I, v, p), V^{p,q}(I, v, p)\} \quad (4)$$

where the superscripts refer to inactivity  $i$ , change price and do not produce  $p$ , produce and do not change price  $q$  and produce and change price  $p, q$ . These options, in turn, are defined by:

$$\begin{aligned} V^i(I, v, p) &= p \cdot E(y') - \alpha \cdot I + \beta E_{v'|v}(V(I', v', p')) \\ &\quad s.t. \quad I' = R(I - y') \\ V^p(I, v, p) &= \max_{\{p'\}} p' \cdot E(y') - \alpha \cdot I - F^p + \beta E_{v'|v}(V(I', v', p')) \\ &\quad s.t. \quad I' = R(I - y') \\ V^q(I, v, p) &= \max_{\{q \geq 0\}} p' \cdot E(y') - \alpha \cdot I - F^q - c(q) + \beta E_{v'|v}(V(I', v', p')) \\ &\quad s.t. \quad I' = R(I + q - y') \\ V^{p,q}(I, v, p) &= \max_{\{q \geq 0, p'\}} p' \cdot E(y') - \alpha \cdot I - F^q - c(q) - F^p + \beta E_{v'|v}(V(I', v', p')) \\ &\quad s.t. \quad I' = R(I + q - y') \end{aligned} \quad (5)$$

As RHS of the functional equation defined by (4) and (5) satisfies the sufficient Blackwell conditions (monotonicity and discounting), these equations implicitly define  $V(\cdot)$  as the (unique) fixed point of a contraction mapping that maps continuous functions into continuous functions, which can be reached by an iteration process given an initial guess for  $V_0(I, v, p)$  (Value Function Iteration).

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<sup>6</sup>For simplicity, it is assumed that the price index is exogenous and known, so it is not another state variables.

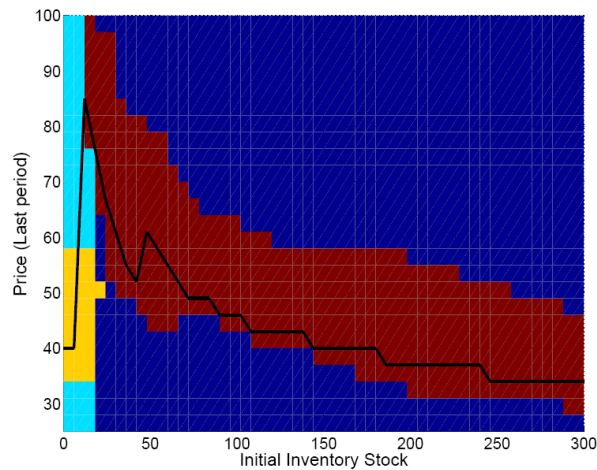
### 3 Results

In this section, we present results of several exercises that will help to understand the firm's optimal inventory, production and price decision and the dynamics of them. To solve the numerical exercises, we approximate the demand shock autoregressive process by a first order Markov process. This allows to simplify the computation of the expected values in the value function iteration approach we follow.

#### 3.1 Optimal decision rule

The optimal decision rule defines a partition of the space  $(I, p)$  for each value of the previous demand shock (the third state variable). Figure 1 presents the mentioned partition and the optimal price in the absence of menu costs (black line). Each zone is identified with one of the four options described in the previous section: inaction (red zone), change price and do not produce (sky blue zone), do not change price and produce (yellow zone) and change price and produce (blue zone). Note that the zones where price is sticky are around the optimal price path without menu costs. The intuition is straightforward: the cost of keeping the price unchanged is proportional to the gap between previous period price (state) and the optimal one (in a no-menu-costs context), so when this distance is small enough, avoiding menu costs is optimal.

Figure 1: Optimal decision rule given a particular realization of the demand shock on the previous period



### 3.1.1 Substitution relationship between available stock and price

The results reveal a negative correlation between the available stock, inventories at the beginning of the period plus production, and price as shows the simulation presented in Figure 4<sup>7</sup>. The intuition behind this result is summarize in the expected sales equation (2) which implies a substitution relationship between the two mentioned variables as noticed in Aguirregabiria (1999). Formally:

$$\begin{aligned}
\frac{\partial E(y_t|v_{t-1})}{\partial q_t} &= \int_{Ln(I_t+q_t)-Ln\left(A.\left(\frac{p_t}{\bar{p}_t}\right)^{-\gamma}\right)}^{\infty} 1.f_{v|v_{t-1}}(v)dv \\
&= 1 - F_{v|v_{t-1}}\left(Ln(I_t+q_t) - Ln\left(A.\left(\frac{p_t}{\bar{p}_t}\right)^{-\gamma}\right)\right) \\
\frac{\partial E(y_t|v_{t-1})}{\partial q_t \partial p_t} &= -\gamma.p_t^{-1}.f_{v|v_{t-1}}\left(Ln(I_t+q_t) - Ln\left(A.\left(\frac{p_t}{\bar{p}_t}\right)^{-\gamma}\right)\right) < 0
\end{aligned}$$

The substitutability arise from the existence of a positive probability of stockout, specifically from its impact in the price elasticity of sales. As the available stock decreases, the sensibility of sales to price also decreases because is more likely that a stockout occurs and, as consequence, it is optimal for the firm to increase its price instead of order. This result is reflected in price and production policy functions (optimal decisions for the control variables given the states) which are presented in Figure 2 and 3, respectively. For a given previous price and three different previous demand shocks we will see, as expected, that the firm finds as an optimal decision to raise its price at the inventory level where orders change from a positive level to zero (when the available stock is the minimum). For inventory levels larger than this critical stock, we may suspect that the optimal price monotonically decreases as initial inventory stock raises (since this implies a larger elasticity of expected sales respect to price). However, as Figure 2 shows, there are other inventory level where the price peaks. The benefit of these price increases is avoiding future orders: when the firm raises its price, its expected sales falls and next period's initial inventory stock increases. For some the inventory levels where optimal price raise, the mentioned increase allows to delay the next order one period.

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<sup>7</sup>The explanation about the differences between both specifications (with and without menu costs) will be presented in Section 3.2.1.



Figure 2: Price policy function

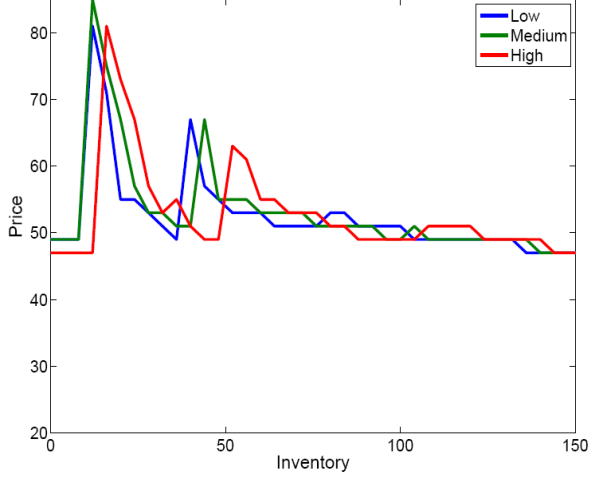


Figure 3: Production policy function

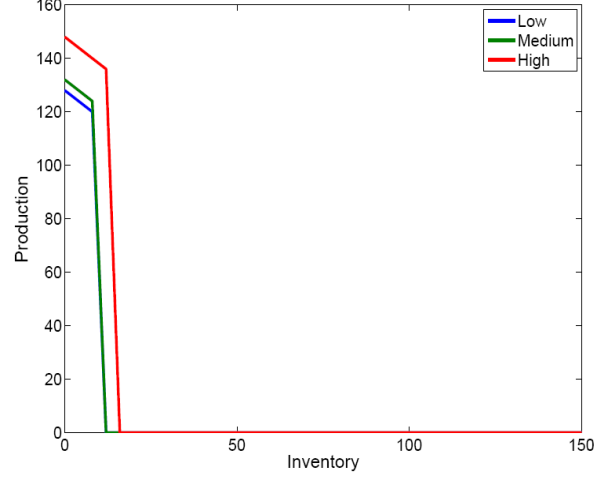
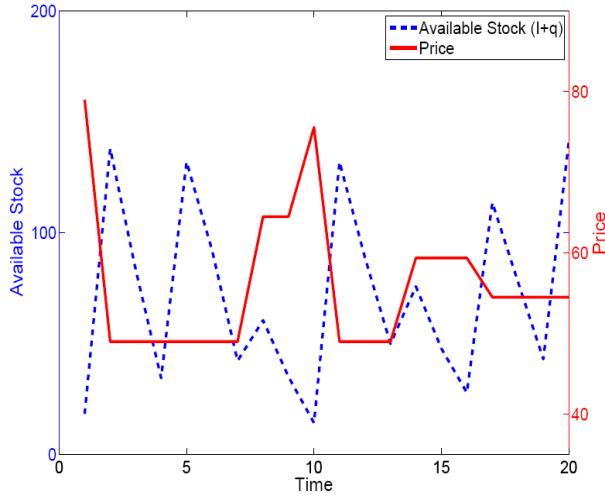
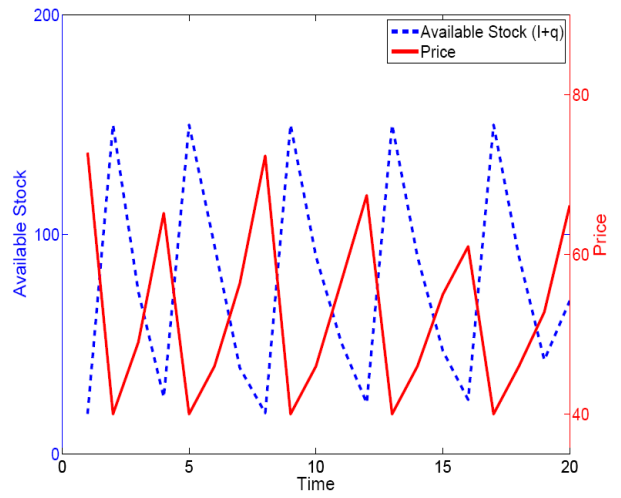


Figure 4: Time series of available inventory stock and price of a firm

4(a): With Menu Costs



4(b): Without Menu Costs



### 3.1.2 The $(S, s)$ rule

The policy function of production reveals that the optimal inventory policy is a  $(S, s)$  rule, an standard result in literature<sup>8</sup>. The value of  $s$  represents a critic value of inventories: if inventory level is above  $s$  the firm decides not to produce, while if inventories fall below  $s$  the firm will produce until its inventory stock reaches  $S$  level. In Figure 3, for example, given a high demand shock,  $s = 18$ . This means that if the firm has  $x < 18$

<sup>8</sup>See, for example, Scarf(1959)

units of inventories, it will produce  $S - x = 148 - x$  to ensure a  $S$  level of inventories. It is important to note that the values of  $s$  and  $S$  may depend on the state variables and strongly relies on the firm's cost structure.

The intuition behind this last point is straightforward: when fixed order costs are more important relative to marginal unit cost, the firm will find optimal increasing its order size (larger  $S$ ) and allow that its inventory stock fall larger before place an order (lower  $s$ ). These allows the firm to make orders more infrequently avoiding fixed costs. As an illustration, Table 1 describes the  $(S, s)$  rule for different values of fixed cost.

Table 1:  $(S, s)$  Rule for different  $F^q$

	$F^q = 100$	$F^q = 300$	$F^q = 500$	$F^q = 700$
$S$	48	90	132	138
$s$	30	12	6	6

### 3.1.3 First Order Conditions

Since solving analytically the first order conditions (FOCs) for the firm's problem presented in Section 2 is not manageable because of the discrete choices it involves, in this section we will evaluate the FOCs conditional to those choices, and discuss the impact of altering some features of the model. That is, we evaluate the FOC that the order size decision must satisfy once the firm has already decided to place positive orders, and the FOC that the optimal price satisfies once the decision of changing price is already taken. For simplicity, we will abstract from the presence of menu costs.

The FOC for the production reveals that the marginal cost of an additional unit of production must be equal to its marginal benefit, which is the price if the unit is sold - the last unit is sold only when the firm stock out-, or the marginal present value that the extra unit of inventory net of depreciation that would generated the next period if the additional unit is stored to be used the next period (Equation 6). This marginal valuation includes the cost of replacement and the valuation of delaying orders.

$$\underbrace{p' \left[ 1 - F \left( \ln(I + q) - \ln \left( A \cdot \left( \frac{p'}{\bar{p}} \right)^{-\gamma} \right) \right) \right]}_{\text{Stockout probability}} + \underbrace{\beta \cdot R \cdot E \left[ \frac{\partial E(V(I', v'))}{\partial I'} \right]}_{\text{Present value net of depreciation of an additional unit of inventories}} \underbrace{F \left( \ln(I + q) - \ln \left( A \cdot \left( \frac{p'}{\bar{p}} \right)^{-\gamma} \right) \right)}_{\text{Non-stockout probability}} = c \quad (6)$$

In the LHS of equation 6, we have that the probability of do not stock out is a positive function of the order size ( $q$ ), and that the marginal valuation of next period's inventories ( $\partial E(V(I', v'))/\partial I'$ ) depends negatively on the order size (conditional on placing orders). The intuition behind the first of these features is as follows: with an additional unit of inventories, a larger demand shock is required to induce the firm to stock out, so it is more unlikely. It is important to clearly understand the second feature, inventories generate value to the firm not only because they can be sold (if this were the case, its valuation would be equal to expected benefit of its sale), but because they allow the firm to place orders infrequently avoiding the lump sum cost. In this sense, at the beginning of the period the unit of inventories number ( $s$ ) is the most valuable since it implies that the firm will not order this period, inventory units before this threshold level do not delay orders so their valuation is only the expected benefit of its sale, while inventory units above the threshold do contribute to delay orders, but the larger the inventories the less important is the last unit (since due to storage and depreciation costs it is not optimal to delay orders too long)<sup>9</sup>. It follows from this discussion that the change in the marginal valuation of next period's inventories is in general negative (except when next period's expected inventories are equal or lower than  $s$ , if they are lower the change is zero, while at the threshold level it is positive), so  $\partial E(V(I', v'))/\partial I'$  must be a negative function of the order size ( $q$ ) for the relevant range.

Figure 5: First Order Condition of production for different values of R and CC

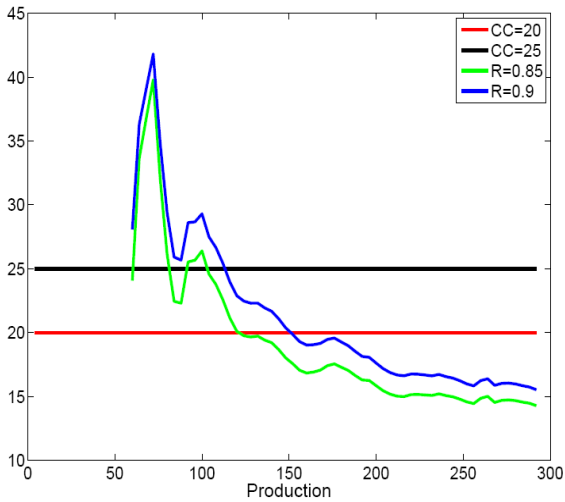
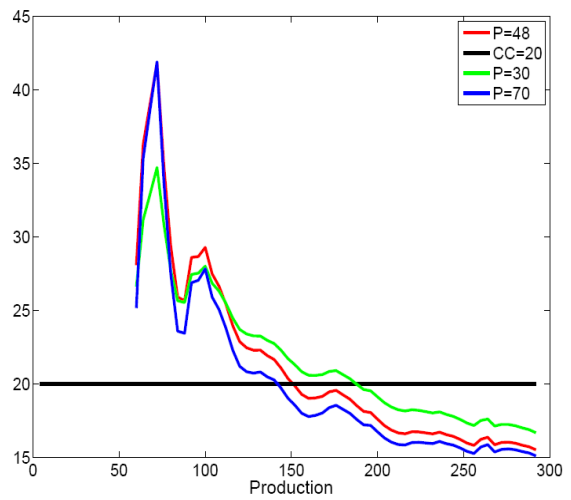


Figure 6: First Order Condition of production for different values of P



Since the change in the probability of stock out probability induced by the marginal change in inventories must be considerably small (out of the threshold levels), we will neglect it, so we can conclude that the LHS

<sup>9</sup>As mentioned before there exist other threshold levels of inventories associated to the inventory levels that allows to delay the order an extra period.

of 6 is a negative function of the order size. With this in mind, it is easy to see the effect of a positive change in the marginal cost ( $c$ ), or in the depreciation rate ( $1 - R$ ) for a given price. Figure 5 shows that both imply a reduction in the optimal order size. Consider now a change in price ( $p'$ ), neglecting the change in probabilities -for simplicity-, the LHS derivative with respect to price is defined in equation 7. Since the expected marginal valuation change is negative ( $\partial^2 E(V(I', v'))/\partial I'^2$ ) and the price increase generates, through lower sales, an increment in expected inventories, the second term is strictly negative. The magnitude of the second term largely exceed the first one, mainly because next period's inventories are quite sensitive to price changes; then the derivative is negative. This implies that a higher price will reduce LHS, and as the depreciation rate shock will lead to lower orders size (See figure 6) confirming the intuitive approximation previously presented.

$$1 - F \left( \ln(I + q) - \ln \left( A \cdot \left( \frac{p'}{\bar{p}} \right)^{-\gamma} \right) \right) + \beta \cdot R \cdot E \left[ \frac{\partial^2 E(V(I', v'))}{\partial I'^2} \frac{\partial I'}{\partial p} \right] F \left( \ln(I + q) - \ln \left( A \cdot \left( \frac{p'}{\bar{p}} \right)^{-\gamma} \right) \right) < 0 \quad (7)$$

Turning to the FOC associated with price (Equation 8), the result indicates that the optimal price is a constant mark-up over the marginal valuation of next period's inventories when a the stock-out probability is zero. When we allow for the possibility of stock out, it becomes possible that the firm's sales do not depend on its price, so it is optimal that the firm raise its price (compared to the case where stock out is not possible). Notice that the second term of the RHS of equation 8, which is associated to the probability of stock out, generates the optimal price to increase. Figure 7 illustrate the last result, note that the complete RHS curve is always above the value of the first term of the RHS (horizontal line), therefore the first one always generates a higher optimal price (the interception with the LHS, 45 degree line). Consider now how the order size decision affects the optimal price setting. Larger orders implies a lower expected marginal value of next periods inventories, so the first term of the RHS of the equation decreases. Larger orders also imply a lower stock out probability, so the effect over the second RHS term depends on the level of the order size, however the effect of larger orders on price decision is given mainly through the decrease in the marginal valuation of next periods inventories, that is the higher the orders size the lower the optimal price, as it is shown in Figure 8.

$$p' = \underbrace{\left(\frac{\gamma}{\gamma-1}\right) \cdot \beta.R.E \left[ \frac{\partial E(V(I', v'))}{\partial I'} \right]}_{\text{Constant mark-up over marginal valuation}} + \underbrace{\frac{(p')^{\gamma+1} (I+q)}{(\gamma-1)A} \frac{\left[ 1 - F \left( \text{Ln}(I+q) - \text{Ln} \left( A \cdot \left( \frac{p'}{\bar{p}} \right)^{-\gamma} \right) \right) \right]}{\text{Ln}(I+q) - \text{Ln} \left( A \cdot (p'/\bar{p})^{-\gamma} \right)}}_{\text{Price increase due to positive stockout probability}} \quad (8)$$

Figure 7: First Order Condition of price for different stockout probabilities

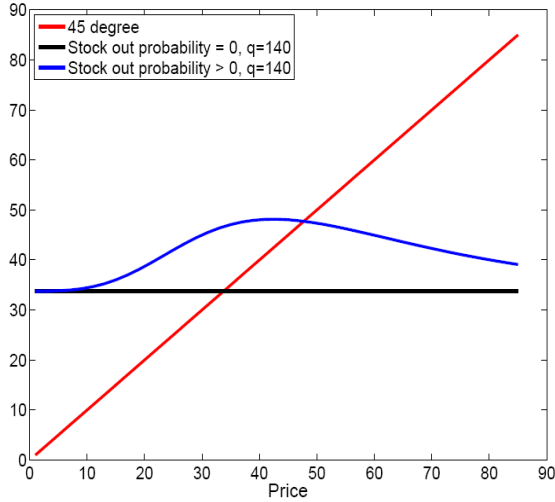
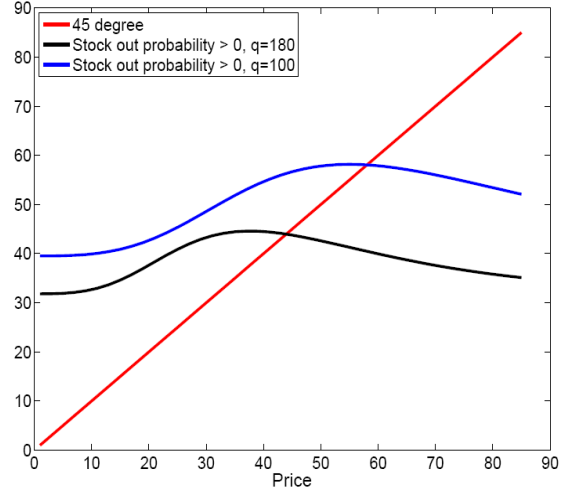


Figure 8: First Order Condition of price for different production levels



It is worth to mention that the FOCs present are quite general in the context of inventory models. For example, consider the version of the model where there is a delay between the order decision and the moment when it arrives (becomes available stock) developed by AKM(2009). In this case, the last unit of production will undoubtedly be carried to the next period (in fact, it will arrive then), so it corresponds to the special case where the stock out probability is zero. As in AKM(2009), the optimal price decision of equation 8 reduces to a constant mark-up over the marginal valuation of inventories, while the order size decision of equation 6 equals the marginal cost only to the mentioned marginal valuation.

To summarize this discussion we should highlight that the possibility of storage goods allows the firm to divide price and production decisions, a potentially important feature to explain firm's behavior, however, those two decisions maintain its fundamental negative relationship.

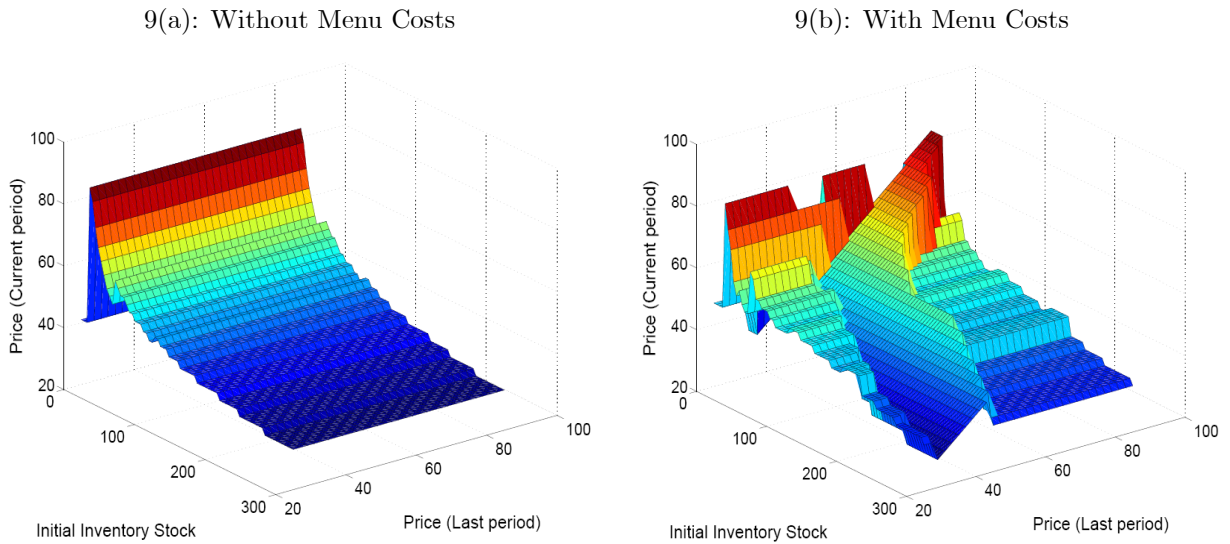
## 3.2 The Workings of the Model

Next, we present some exercises with different model specifications in order to built some intuition about the workings of the model, and to contrast the different responses of cross-section and time series data to each aggregate shocks . We begin by testing the contribution of menu costs in the model in order to understand its role in firms decisions, then we characterize the optimal pricing and inventory behavior when firms face an unexpected change in their cost structure, and an unexpected demand shock. Finally, we compare the responses to the mentioned shocks. We highlight the role of cross sectional information (particularly, the distribution of inventories across firms) to explain the aggregate responses of the relevant variables.

### 3.2.1 Menu costs

Menu costs are often introduced in order to account for the cross section price dispersion and the stickiness of prices observed in the data, however little has been said about its impact on production, sales and inventory stock. We explore these features finding some suggestive results. First, to illustrate the obvious consequence of introducing menu costs -price stickiness-, we simulate the behavior of a firm with menu costs and without them for the same path of demand shocks and initial inventory stock (see Figure 4). As consequence of the stickiness in price, the (negative) correlation between price and available stock is reduced from -0.9 to -0.58. Figure 9 presents the difference between the price policy function with and without menu costs. As the reader must noticed the main difference between these policy functions is the presence of a flat surface on the second figure, which reflects the zone where it is optimal to maintain last period price.

Figure 9: Price policy function change



Note that the presence of menu costs not only guarantees the stickiness of price adjustments, but also has impact on order decisions since both variables are determined simultaneously. Without the introduction of the mentioned friction, the firm's optimal decision can be characterized in two steps: evaluate the optimal price for each production decision, and choose the combination that reports the highest present value of benefits. Hence, it is not possible that the firm set a price not optimal given the production. With menu cost, this feature is no longer preserved: for certain regions in the space of state variables, specifically where previous price is in the neighborhood of the optimal one, the firm will choose the optimal production taken as given that its current price equals its previous period's price in order to avoid the menu cost. In this area, the optimal production level is a decreasing function of last period's price (Figure 10). Because of this, the dispersion of the orders conditioned to positive ones is larger when menu cost are introduced (see Figure 11).

Figure 10: Policy functions given a low initial inventory stock

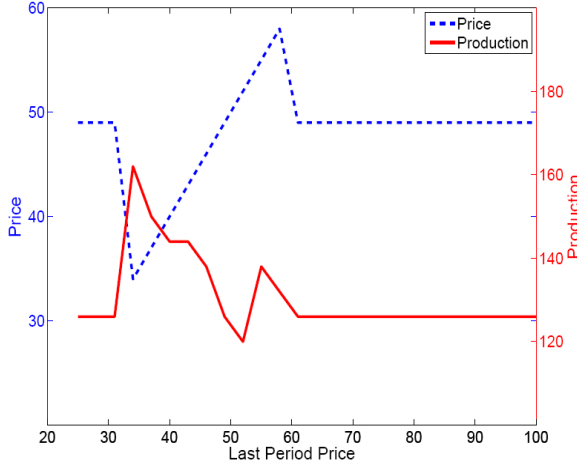
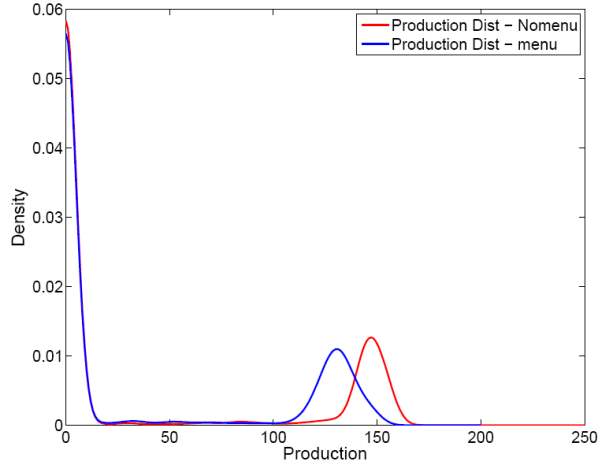


Figure 11: Cross section Production Distribution



Another interesting result is that it is possible that the presence of menu costs reduce cross section variability of prices instead of exacerbate it. At first glance this result seems counter intuitive, but the answer could rely in the substitutability between production and price in the model: since adjusting prices is costly but change the order size is not, the firm may find optimal to adjust its order size to avoid changing its price. Under the parametrization used, this last point implies a reduction in the order size, so the cost of this measure is a higher probability of stock-out (which is reflected in lower expected sales). This might be the intuition behind the fact that price does not change even when orders took place (strong increases in available stock) as can be seen at Figure 4.

### 3.2.2 Unexpected and permanent lump sum cost change

For this and the following exercises, we analyze the effects on cross section and time series data of specific permanent and unexpected shocks. To simulate the model, we use as initial inventory distribution a draw from the invariant distribution, and generate independent series of demand shocks which follow a stochastic first order Markov process. We evaluate changes of 25 percent in each of the key parameters.

Assume that the firms experiment a cost structure change, particularly the lump-sum cost of orders increase 25 percent. As expected, the optimal firm's response to the shock is to place orders more infrequently. To accomplish this, the firm will increase its orders size (higher  $S$ ) and reduce the inventory level that triggers positive orders (lower  $s$ ). So, when the cost shock is realized, some firms which had produced given its inventory level will now decide to wait until its inventory stock reach lower levels. However, firms that effectively produce this period will produce a higher quantity than the one they would have produce in the absence of the cost shock. In terms of the FOC for production, larger order size responds to the raise in the marginal valuation of an additional unit of inventories. Recall that this marginal valuation includes the valuation of delaying future orders which increases as the lump sum cost raise. The described behavior is captured by the change in the production stationary distribution (Figure 12). Despite the increase in order size conditional to positive ones, aggregate production do not necessarily increases when we consider the change in the stationary distribution, because the fraction of firms placing no orders increase. However, in the short term, response of aggregate production must be positive since each firm needs to place a new large order before reducing its order frequency (Figure 14).

Price decision of an individual firm -as function of the initial inventory level- increases as consequence of the mentioned raise in marginal valuation, and peaks at a lower inventory level due to the decrease in  $s$ . However, to analyze the movements in the average price is fundamental to consider the change in the inventories distribution since there exists an important dispersion of prices across inventory levels. The reason for this dispersion is that the expected marginal valuation of inventories decreases as available stock increase. Then, an inventory distribution with higher density at very low and very high levels of inventories (see the price policy function to confirm these inventory levels are associated to lower prices) will generate a lower average price than another inventory distribution (perhaps with the same average inventory levels) which density is concentrated in intermediate inventory levels. The lump sum cost shock generates that the inventory distribution shift to higher inventory levels due to larger orders affecting negatively average price. So, we have two opposite forces driving the average price: higher marginal valuation for delaying orders, and higher inventory levels. In our exercise, average price increase as consequence of the shock, so average sales drop.

The mentioned qualitative long term effects are robust to the presence of menu costs, however the mag-



nitude of these effects and short term dynamics are quite sensitive to them. If menu costs are not taken into account, the adjustment of the average price is automatically, so as the decrease in average sales. Meanwhile average order's size almost does not fluctuate before stabilize. For contrast, when menu costs are considered, average price increase is smooth (so as sales), while orders size fluctuate strongly and takes almost 20 periods to stabilize (Figure 14 and 15). This results suggest that firms delay price movements by adjusting order size.

Figure 12: Production stationary distribution with menu costs

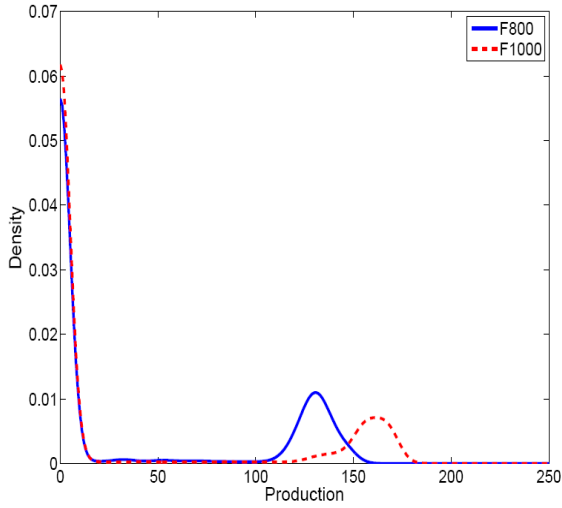


Figure 13: Mean inventories

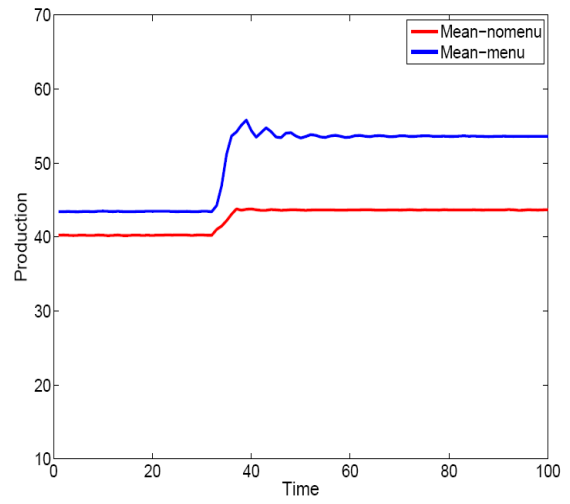


Figure 14: Production mean

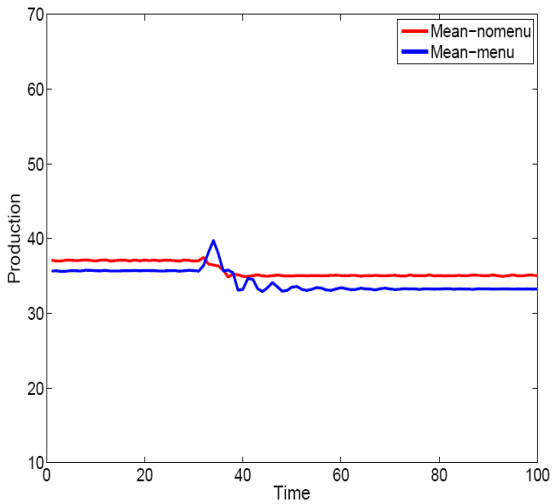
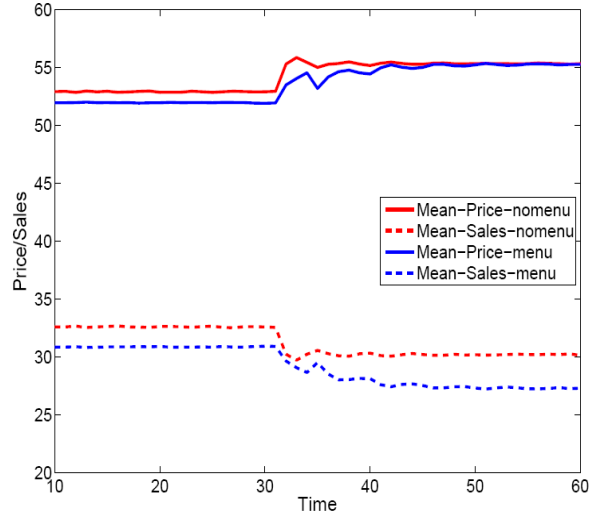


Figure 15: Price and Sales mean



### 3.2.3 Unexpected and permanent marginal cost change

Consider an unexpected increase of 25 percent in the variable cost. Unlike the lump sum cost, variable cost directly affects the marginal cost, so changes generated by this shock would be quantitative larger than the ones presented before. In the production decision, given the increase in the marginal cost of producing an additional unit, the marginal benefit should also increase in order to satisfy the FOC, so since the value of an additional inventory unit is a decreasing function of the production size, the order size must decrease (lower  $S$ ). Turning to price decision, the increment of the marginal cost generates that the marginal valuation of an additional unit of inventories raise for each inventory level, so optimal price (as function of inventories) must increase. This, in conjunction with the fact that right tail of the inventory distribution -the one associated with low prices- disappear, translate into a more than proportional increase in the average price. As a consequence of the large price increases, sales drop sharply.

The reasons for holding inventories are weakened by the marginal cost shock: stock out avoidance motive is reduced because uncertainty about sales is diminished given the important increase of price, and the lump sum cost which inventories allows to avoid is now less relevant compared to the variable cost. Then, inventory holdings decrease, particularly, as mentioned, the right tail of the distribution disappear because of the reduction in the size of orders.

Figure 16: Price and Sales mean

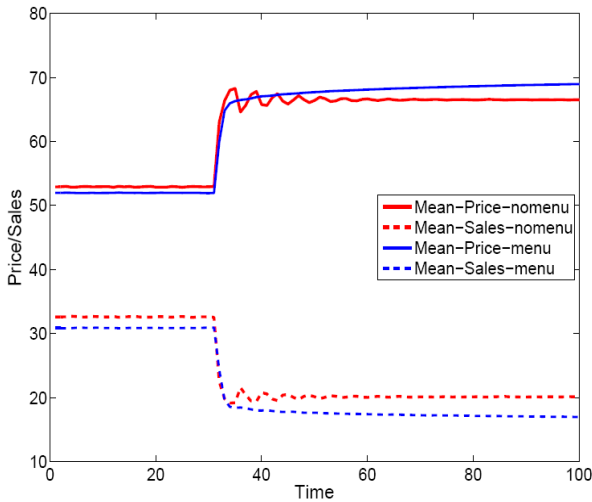


Figure 17: Production mean

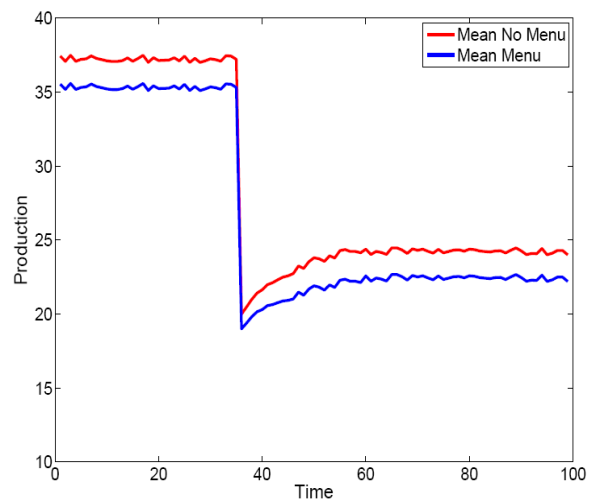


Figure 18: Production stationary distribution with menu costs

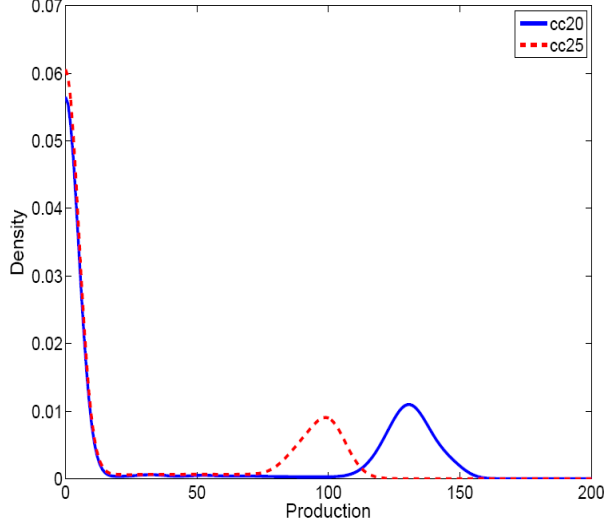
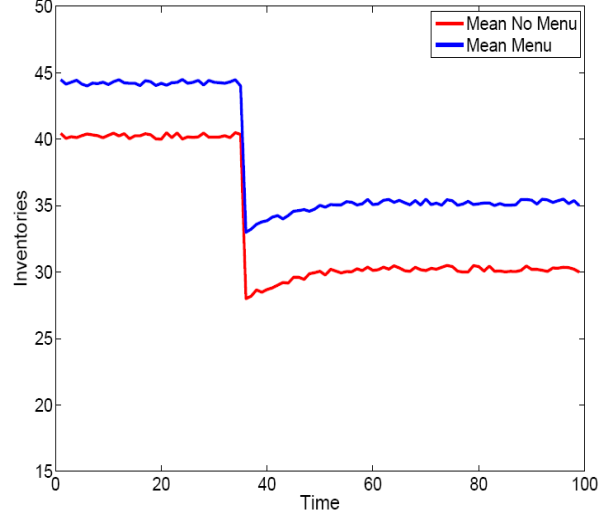


Figure 19: Mean inventories



### 3.2.4 Unexpected and permanent demand shock

This subsection discusses the effects of 25 percent reduction on the demand. The reduction of demand decrease the probability of stock-out, and since it is expected to be permanent , it also reduces the marginal value of next period additional unit of inventories. The intuition behind this reduction is that a permanent negative shock to demand reduces the price at the firm will sell the good independently of the period when the sale occurs. Recall equation 6, for a given price the negative demand shock implies a reduction of the LHS, so it reduces the optimal order size (lower  $S$ ). This is captured by Figure 20 which shows the change of the stationary distribution of production. Meanwhile, from the FOC for price, optimal price for a given production level decreases as consequence of the reduction of marginal valuation and the probability of stock out. This because the constant mark-up is applied over a lower value, and the lower stock out probability make sales more sensitive to price. However, the result of our exercise is that average price increases as shown in Figure 21. The explanation relies in the reduction of the order size and the change in the distribution of inventories. The first feature implies an increase in the stock out probability and in the marginal valuation of an additional unit of inventories (both increase optimal price), while the reduction of the right tail of the distribution of inventories also contribute to raise average price.

Note that uncertainty over the shocks continues only that it turns on smaller possible quantities weakening the stock out avoidance motive to store inventories. Meanwhile, sales sharply drop because of the demand shock and the price increase (see Figure 21). Despite the large decrease in sales, inventory holdings reduces

as a consequence of the smaller order size. Note that frequency of orders does not change since the proportion of firms which do not place orders is the similar before and after the shock (see Figure 20). The described results are robust to the presence of menu costs, which are reflected on the more softly trajectory of the price and sales to their new mean after the demand shock.

Figure 20: Production stationary distribution with menu costs

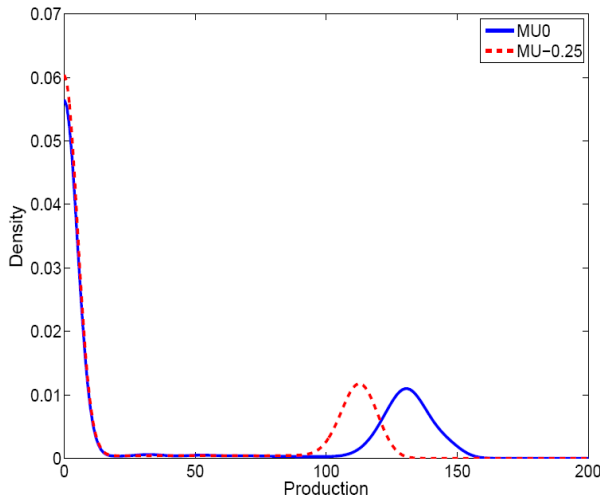


Figure 21: Price and Sales mean

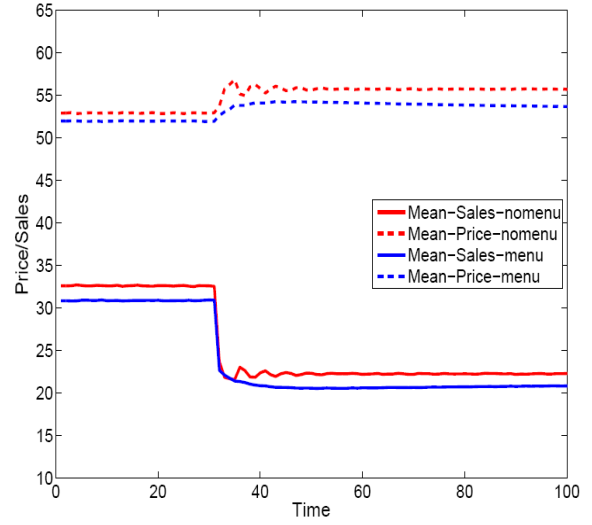


Figure 22: Production mean

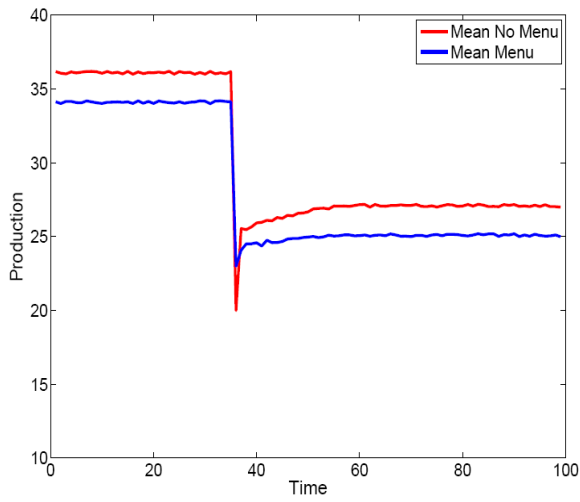
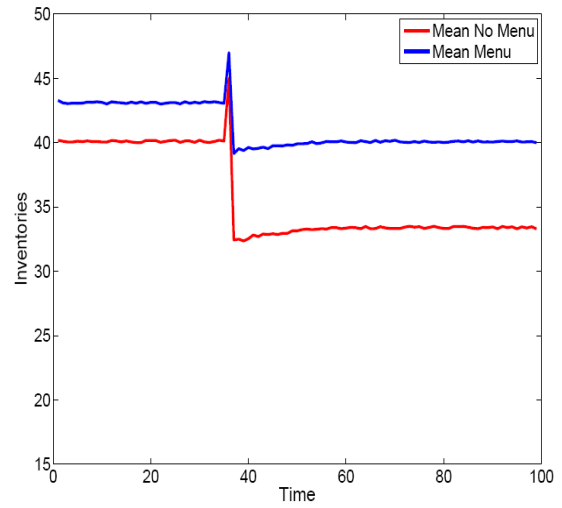


Figure 23: Mean inventories



### 3.2.5 Comparing aggregate shocks

Previous analysis describes the individual responses of main variables to each aggregate shock. Now we compare them taking into account that shocks' size is not identified, which implies that quantitative responses are not comparable between shocks. However, it is possible to compare quantitative responses within a shock. Therefore, we highlight qualitative differences in long term effects (changes in cross-sectional stationary distributions), and short term dynamics.

We begin with long term effects. Table 3 presents the changes respect to the base scenario of the stationary distributions statistics for principal variables. Consider inventories, sales and production. Since inventories change is the difference between production and sales (neglecting depreciation), it might be confusing that, despite that the drop in sales is larger than the decrease in production, inventories fall as consequence of the demand and marginal cost shocks. Besides, under lump sum cost shock, the increase in inventory level is considerably high if we want to think that this arise only from the more than proportional reduction in sales respect to that of the orders. To explain these features, it becomes important to remember that we are considering change in stationary distributions which may be different from the short term dynamics, and to understand that the average production is not necessarily informative about the level of inventory holdings. A simple example might help. Imagine that there are three firms and each one sells one 100 products at an exogenous price. Consider two cases: in the first, each firm produces 100 every periods, and in the second one, each firm produces 300 every three periods. The mean of aggregate production is the same every period, however, in the first case the mean of inventories at the end of the period is zero, while in the last case, it is 100.

Table 2: The  $(S, s)$  rule for different shocks

	Base scenario	F1000	cc25	Mu -0.25
$S$	128	156	100	112
$s$	12	8	8	8

As the reader must noticed, the explanation of the inventories distribution movements heavily relies in order's frequency and size. So, the impact of each shock on the  $(S, s)$  rule is fundamental to characterize inventory behavior (see Table 2). Since in the case of the lump sum cost shock orders size raise, inventory holdings increase. Meanwhile, cost shocks reduce order size so inventories decrease.

Table 3: Identification strategy

		Fixed costs		Marginal costs		Demand	
		ST	LT	ST	LT	ST	LT
Inventories	Mean	↑	↑	↓	↓	↑	↓
	Variability	↑	↑	↓	↓	↓	↓
	Right tail (p90)	↑	↑	↓	↓	↑	↓
Production	Mean	↑	↓	↓	↓	↓	↓
	Variability	↑	↑	↑	↑	↑	↑
	Right tail (p90)	↑	↑	↓	↓	↓	↓
Price	Mean	↑	↑	↑	↑	↑	↑
	Variability	↑	↑	↑	↓	↑	↓
	Right tail (p90)	↑	↑	↑	↑	↑	↓
Sales	Mean	↓	↓	↓	↓	↓	↓
	Variability	↑	↓	↑	↓	↓	↓
	Right tail (p90)	↓	↓	↓	↓	↓	↓
Mark-up	Mean	↑	↑	↓	↑	↑	↑

Large decreases in price variability as response to demand and marginal cost are worth to explain. Notice that the price variability reduction is directly related to the increase in optimal price and an upper limit to the price that does not vary between the different scenarios. This last feature is associated to the demand function, under any shock the firm does not consider optimal to choose a high price as it would generate to hold inventories for larger periods and incur in higher depreciation and storage costs.

So far we have focused on the main statistics of the distributions cross section for each case, however, short term dynamics can be as informative as changes in stationary distributions, therefore we now discuss the most informative short term dynamic of the principal variables. The dynamics of the variables not presented move to the same direction since the shock is realized until the next steady state.

As the reader might anticipate, average production immediate response to a lump sum cost shock (a sharp increase) is exactly the opposite from this variable dynamics in response to the other analyzed shocks, even though the average production in the stationary distribution falls as consequence of the three shocks. The reason is that the lump sum cost shock, unlike the other ones, implies a higher optimal production decision (larger order size), so the average of production increase, before lower orders frequency generates a reduction of it as can be seen in Figure 25.

It is also worth to mention the overshooting in the mean and variability of the mark-up (also you can verify that with the percentiles 10 and 90) immediately after the marginal cost shock. This occurs because firms set the price of its good before learning the shock. The path shows that mark up variability increases before adjusting to its new steady state level which corresponds to a lower than the initial one (See at Figure 26).

Figure 25: Time series of mean production under three scenarios

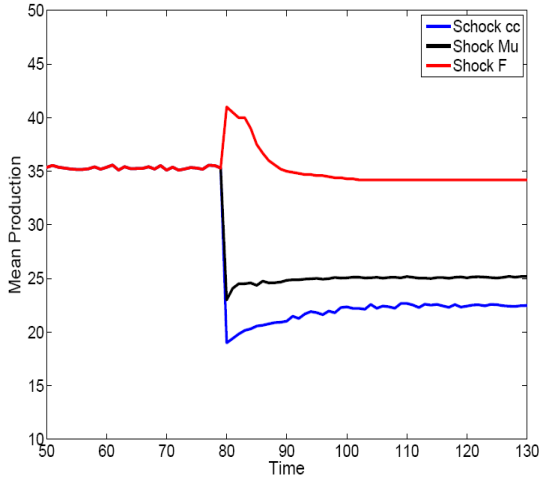
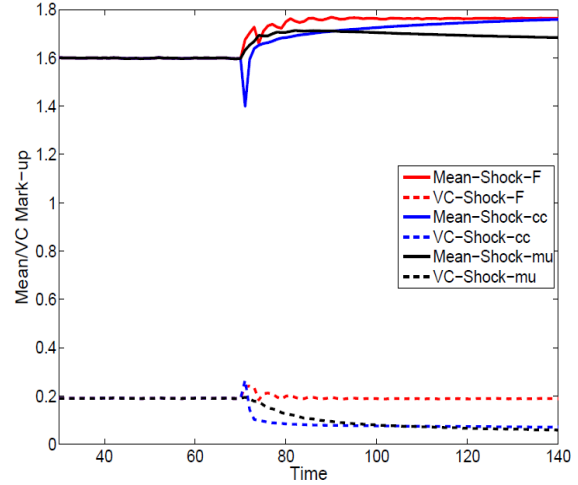


Figure 26: Time series of mean and variability of mark-up under three scenarios



Even though shock's size is not identified, large differences between price responses after a demand and a marginal cost shock is worth to comment, specially because according to previous section discussion the price movement in response to a demand shock is ambiguous. Price increase is significantly larger in the case of a marginal cost shock. The reasons are the opposite movements in expected marginal valuation of inventories, and the quantitative difference in the reduction of inventory holdings. Marginal valuation of inventories decrease with the negative demand shock in line with the expected lower demand, while it increases when marginal cost raise as consequence of the increase in the replacement cost. In the case of the lump sum cost shock, price average slightly increases (3 percent) because the effect of larger marginal valuation is offset by larger inventory holdings.

## 4 Empirical analysis

The last financial crisis of 2007 - 2009 caused important movements in firm's decisions and outcomes through different channels, assuming firms operate as the simple inventory model we have proposed, we will try to identify the main channel through which the last international crisis was transmitted to the manufacturing and trade sectors (the ones for which an inventory model might be more relevant). The strategy will be to compare the responses analyzed in previous sections to data movements. Results will be suggestive since available information is insufficient to calibrate the model.

### 4.1 Data description

We use information for two countries, the one in which the crisis was originated, United States (US), and a developing country commercially integrated to it, Peru. An important limitation of both datasets is that we only have access to financial reports, thus we cannot distinguish directly if a movement in a particular variable is a response to a quantity or price change. To deal with this, we construct ratios that pretend to cancel out prices and allow us to identify relative movements in main variables.

#### 4.1.1 Peru

The data we use is from the Annual Economic Survey, which is collected by the National Institute of Statistics and Information (INEI). The survey is conducted every year since 1976 to firms which sales exceed an specific amount<sup>10</sup> and to a random sample of smaller firms. The survey covers approximately 27,000 firms including manufacturing, and trade sectors. However, we only had access to information of the firms for the 2007-2009 period, and only of firms which report information consistently during the period (therefore, it is balanced panel).

The survey includes information about financial reports but we will use only three of them: balance sheet and income statement by function and nature). They contain information about inventories, sales and cost of sales that allow us to distinguish if the firm is buying merchandise to sell them again (trade activity) or if it use inputs to elaborate products (manufacturing activity). We only consider variables associated to the manufacturing (trade) activity for the firms in the manufacturing (trade) sector, in order account for the differences these activities.

For the main variables in levels, we construct variables adjusted by the mean (across time) in order to avoid having results driven by firm size, a feature ignored in the proposed model, so ratios have no need for

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<sup>10</sup> Approximately US\$ 370 thousand



adjustment. Finally, for all variables we drop observations that are clearly an outcome of errors and outliers that could disturb results. Our sample consists of about 596 manufacturing (from which 125 are textile ones) and 1,886 trade firms over a three year-period (2007-2009). Taking into account this temporary limitation in available Peruvian data, we will try to identify movements in data through the statistics changes of the main variables distribution.

#### 4.1.2 United States

We use Compustat quarterly information from 1990Q2 to 2010Q3 . The survey provides data of sales, purchases, inventories, costs, and others but in this case, we could not access to disaggregated data of these main variables. Then, we could not difference between products and merchandises. As in the Peruvian case, the information allows us to construct measures and ratios according to the model described above. But, since we have long panel, we were able to deal with the cycle and firm size issues by calculating the porcentual deviation from trend from the seasonally adjusted variable, where the trend was captured by a HP filter.

After excluding observations that are an obvious outcome of error, dropping outliers, and the requiring that an included firm reports at least three years of information, we are left with 224,265 manufacturing and 47,242 trade plant-quarter observations.

## 4.2 Empirical Correlations

Before turning to the discussion of the main transmission channels, it is important to highlight that assuming that manufacturing and trade firms work as the simple model presented is clearly an over simplification. However, a simple correlation analysis suggests that some stylized facts of U.S. data<sup>11</sup> match with some of the principal results of our model. We calculate the relevant correlations using the porcentual deviations of the aggregate mean (seasonally adjusted) levels respect to the trend calculated with the HP filter.

If we consider only the aggregate information of the main variables (see Table 4)<sup>12</sup> it might seems that firms' behavior is far from the one predicted by our inventory model. However, if we look at cross-section information, we find that individual firm decisions resemble most of its principal predictions. Table 5 presents the average of the individual firms time series correlations (as opposed to the correlation of the aggregate variables presented in Table 4) and cross-sectional variability. These figures are aligned with the main predictions of the model: the negative correlation between price and production decisions, as well as the negative correlation between price level and its variability, and the negative correlation between the right tail of the distribution of inventories with price. But evidence does not suggest other predictions like the

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<sup>11</sup>The correlation analysis is performed only with U.S. data because of the low frequency and short period available for Peruvian information.

<sup>12</sup>In tables 4 and 5 we use mark-up instead of price, because the available information does not allow it.

negative correlation between sales and price, and negative correlation between inventories at the beginning of the period and orders size. It is also worth to mention that the model predicts that sales response to shocks is larger than inventory response, so the inventories to sales ratio is countercyclical which is an standard stylized fact.

Table 4: Stylized facts in U.S. aggregate data

Correlation	Trade	Manufacturing
Sales,Mark-up	0.32	0.25
Initial Inventories,Production	0.74	0.48
Production,Mark-up	0.39	0.27

Table 5: Stylized facts in U.S. cross sectional data

Correlation	Trade	Manufacturing
Sales,Mark-up	0.03	0.04
Initial Inventories,Production	0.34	0.27
Production,Mark-up	-0.10	-0.09
Inventory_P90,Mark-up	-0.43	-0.50
Inventories_VC,Mark-up_VC	-0.15	0.36
Mark-up,Mark-up_VC	-0.01	-0.81

We also performed a rough empirical approximation of the FOC for price by a regression of the mark-up over initial inventories, production and sales. Since the marginal valuation of inventories is a negative function of inventories and production, we expect negative signs for those coefficients, while, if we consider that firms expectations of sales is quite accurate, the sign of its coefficient should be positive since larger expected demand affects positively the valuation of an additional unit of inventories. Results confirms these hypothesis.

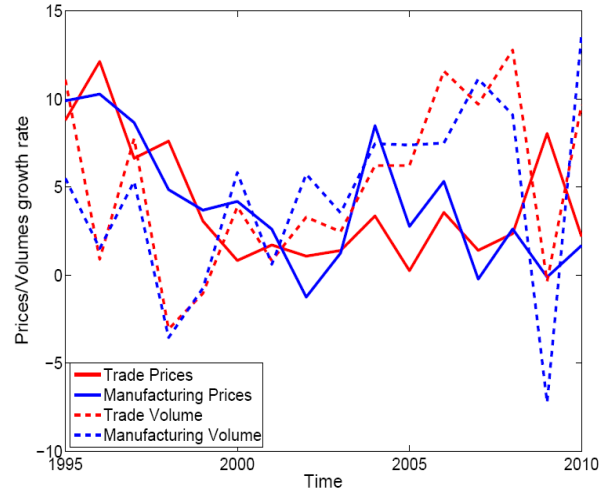
### 4.3 Identifying main transmission channels

We now move to identify the main transmission channels through the comparison of data responses with the exercises presented in in the previous section. The idea is to suggest which might be the main channel of transmission for each sector within each country.

### 4.3.1 Peru

Between 2002 and 2008, manufacturing and trade sectors experienced an important acceleration in growth rates. For manufacturing, the growth rate step from 5.7 percent in 2002 to 9.1 percent in 2008, while trade sector rise from 3.3 percent to 12.8 percent. However, in 2009, after the international financial crisis became sharpen, both sectors stopped their expansions: manufacturing and trade faced contractions in their production volumes of 7.2 percent and 0.3 percent, respectively. A year later both sectors recovered, and they reached growth rates greater than 9 percent (see Figure 27).

Figure 27: Trade and Manufacturing  
peruvian time series



Source: INEI

Although two sectors experienced important reductions in production volumes during 2009, the heterogeneity of the firm behavior of each case is evidenced from the different reactions of the other variables such as price. Manufacturing price index fell but not significantly (-0.1 percent), while for the trade sector the effect was the reversed: its price index accelerated (8.0 percent).

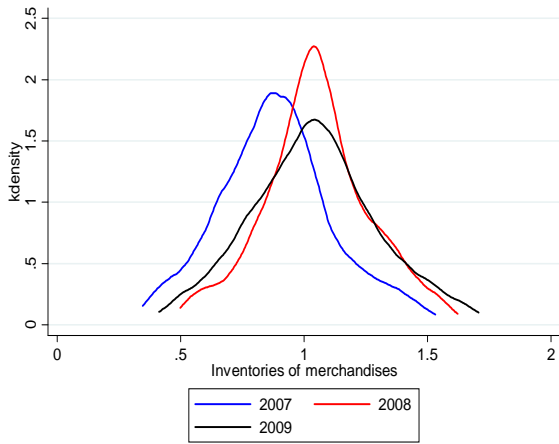
Within manufacturing sector, textiles faced the largest movements in data: a drop of 6.6 percent in 2008 and 24.6 percent in 2009 after a sustained growth in previous years. Even the recovery was more pronounced than in manufacturing or trade: 36.5 percent growth for 2010. Meanwhile its price index fell in 2009 but also not significantly (-0.7 percent). In order to see the sense of the great data movements in manufacturing, when we treat it we will consider the specific case of textiles.

This first approximation to the Peruvian aggregate data of the manufacturing and trade sectors, leads us to believe that the last international financial crises (2007–2009) would affect to each one by different

channels. For this, under a more detailed analysis of time series and cross section data we find evidence that suggest which is the most important transmission channel for each sector.

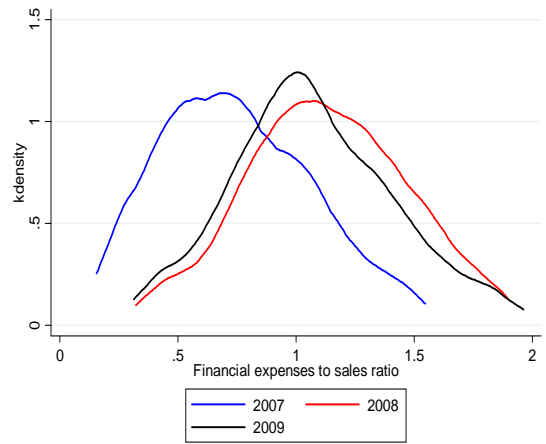
**Trade sector** As the previous discussion of aggregate data pointed out, micro data verifies the positive performance of the sector until 2008. Sales, purchases, and inventories grew from 2007 to 2008, and decreased for 2009. Meanwhile, mark-up slightly increased in the 2007-2009 period, and its variability reduced for 2009 after increasing in 2008. Merchandise orders increased its dispersion in 2009. As mentioned before, this results can involve changes in volumes and/or prices. To disentangle this, we observe some informative ratios, such as the sales to inventories ratio, which indicate that in 2007–2008 the increase on sales was larger than inventories and the pronounced fall in sales explain the reduction on this ratio for 2008–2009. Since orders are valued at cost, we can ascribe the last reactions to effects in volumes assuming marginal cost is constant over the period. This allows us to identify increases in volume of sales and orders for 2007–2008 and reductions for 2008–2009. Besides, volume of sales to volume of orders ratio suggests that the reactions in orders are more pronounced than in the sales for both terms.

Figure 28: Stationary distribution evolution of merchandises inventories



Source: INEI

Figure 29: Stationary distribution evolution of financial expenses to sales ratio



Source: INEI

If firms that belong to the trade sector will take their decisions as in the inventory model proposed, we could infer that the movements in data for 2008 previously described were generated by a shock that resembles the lump sum cost shock. A possible channel is through financial expenses, which increased more than sales in 2008 and 2009 (see Figure 29).

We propose that in 2008 the crisis affected this sector increasing firms' financial expenditures, which might

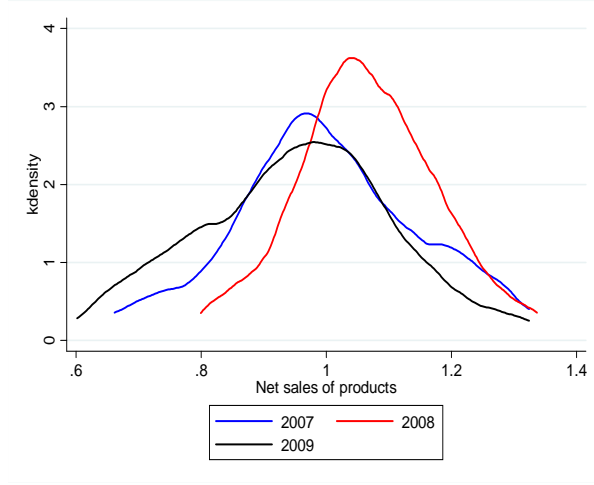
be interpreted as a fixed cost. In consequence, firms took larger orders if they decide to order but the frequency of them does not reduce necessarily because prices did not change yet. Then, inventories rose that year. In 2009 firms decide to adjust their prices to higher values and hence sales fell significantly. Orders size increases to avoid the larger lump sum costs, but the frequency of them faced a reduction (this could be appreciated in the inventory days increment and on the increase of orders dispersion). Therefore, the effect on the inventories is not clear in this period.

**Manufacturing sector** For this sector and particularly for textiles firms, net sales of products increased in 2007 – 2008, but reduced for 2009, while sales variability move in the opposite way (increase for 2008, and decrease in 2009). We have to highlight the reduction in foreign sales and specifically the drop of foreign sales of final goods in 2008 - 2009. Considering that foreign sales represent 30 percent and 50 percent of total sales of products for manufacturing and textiles, respectively, we can attribute the drop in sales for 2008 – 2009 as a fall in volumes. So, these movements in sales can explain both the slightly increase for 2007 – 2008 and reduction for 2008 – 2009 in cost of sales.

The effects on the mark-up reveal that it experienced positive but not significantly disturbs for both periods and almost imperceptible increase on its variability. This could be positive correlated with the effects on price. Production faced the same movements as sales on average and on its concentration. However, volumes of sales to volumes of production ratio suggest that the effects on sales are more pronounced on average. Inventories (without merchandises to be more strictly) rose significantly with a higher concentration for the first period but they experienced a little reduction and considerable increase in variability for 2008 – 2009. Sales to inventories ratio indicates that the increase in stocks for the first period is larger than in sales while the decrease on them is lower than on sales for 2008 - 2009.

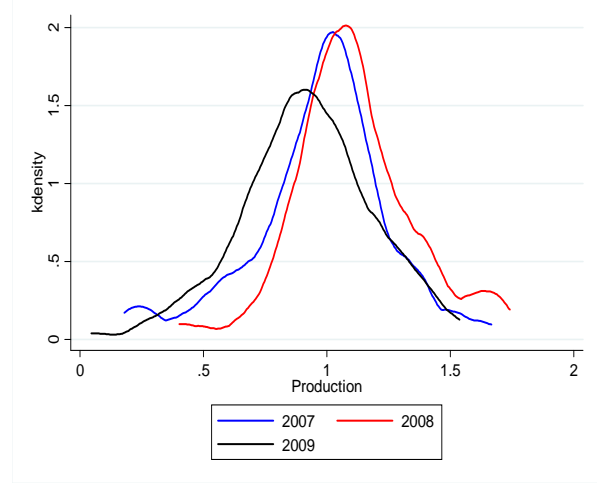
Finally, we cannot conclude about the evolution of financial expenses because, more of the manufacturing/textiles firms are also dedicated to buy and sell merchandises. Then, we cannot identify which part of the financial costs are generated by manufacturing or trade activities. This problem does not happen in trade and this is why we could establish relationships and results in previous subsection with this variable and others related to it.

Figure 30: Stationary distribution evolution  
of sales



Source: INEI

Figure 31: Stationary distribution evolution  
of production



Source: INEI

Under the model assumptions and if we want to think that the firms take their decisions as in the proposed approach, given the last statistical description, we can suggest that a negative demand shock in 2008 is the channel through the international crisis affects this sector. The reduction of inventories, and the slightly movement of mark-up, as well as the importance of external demand in the sector, influenced this conclusion.

#### 4.3.2 United States

**Trade sector** During the period before the sector was affected by the economic crisis (2006Q2-2008Q3), it exhibited positive and increasing growth of average sales, purchases, and inventories; meanwhile mark up was falling since 2007Q1. In 2008Q4, just when the economic crisis worsened, sales, purchases and inventories growth rate collapse, and the mark up kept falling. Then, we will consider that the shock occurred in the mentioned quarter. After the shock, firms decided to keep lowering their purchases, and their mark-up, however soon later while purchases and sales were still falling, firms started to set higher mark-ups (see Figure 32). It is important to notice that inventories do not decrease, but stop increasing during the crisis.

Figure 32: Mean dynamics (dev. from trend)



Source: Compustat

Figure 33: Mark-up dynamics



Source: Compustat

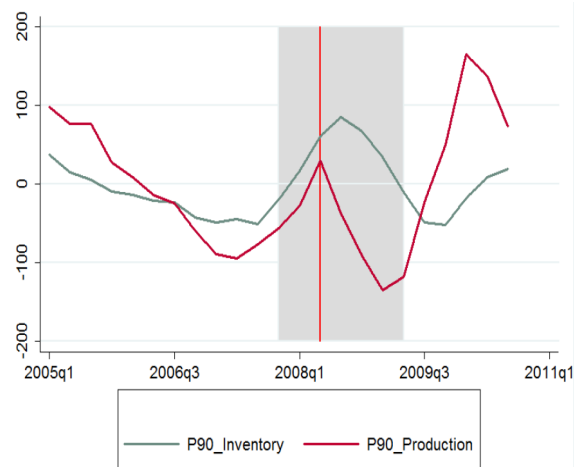
The previous information suggests that the main channel through which the crisis affected the trade sector was through a shock in the variable cost. Recall the discussion about the consequences of this shock: inventories, sales and production drop while price sharply increase, so that the new mark-up is higher despite the marginal cost increase. However, the short term dynamics showed that before reaching its new stationary level, the mark-up sharply decreases as consequence of the unexpected increase in marginal cost, this response is the same that can be seen in Figure 33. Another feature is that our model reproduces this sector evidence is the sharp decrease in mark-ups variability that took place after the shock, as well as the decrease of the 90th percentile of the inventory distribution due to the price increase. If we accept the interpretation, we should now think how this marginal cost might be interpreted. One alternative, which is aligned with the data, is the working capital channel. In the data, working capital sharply increased after the worsened of the economic crisis, meanwhile payable accounts to suppliers dropped. Possibly, as interest spreads widen and liquidity dry out, firms had to replace suppliers' credit (low interest cost) with working capital investments (highly costly given the financial crisis).

Figure 34: Variability dynamics



Source: Compustat

Figure 35: Right tail dynamics

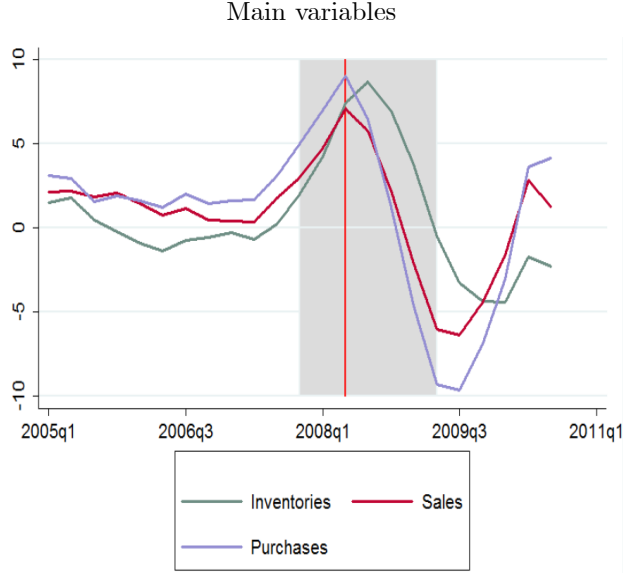


Source: Compustat

**Manufacturing sector** The period previous to the crisis, the development of the sector resembles the one described for the trade sector with sales, inventories and production rising at increasing growth rates, but, unlikely the trade sector, average mark-up were not decreasing. The response of the principal variables (sales, inventories, production, and mark-up) is isomorphic to the one discussed for the trade sector with two differences. First, movements are larger and recovering slower, second the period in which sales, inventories and production experiment a large drop is 2009Q1, not 2008Q4. We think that the explanation for the last feature is that production decisions and sales arrangements are made more in advance due to the delays in the production process, and that the shock realization was indeed in 2008Q4. In this quarter costs rise but since sales arrangements were already made, prices could not change and mark-up sharply decreases. As the reader must suspect, the evidence suggests that the main transmission channel is through increment of marginal costs. Data movements also support a working capital channel cross sectional variability evidence as in the previous case match the response that the model predicted.

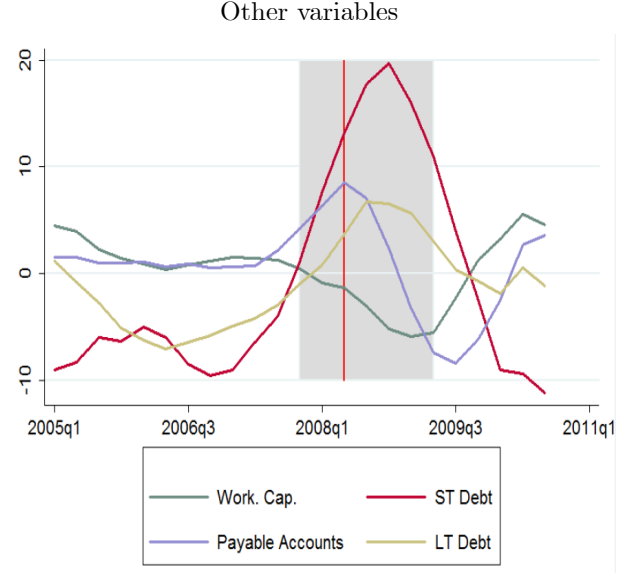


Figure 36: Mean dynamics (dev. from trend)



Source: Compustat

Figure 37: Mean dynamics (dev. from trend)



Source: Compustat

## 5 Conclusions

We study firm's price and production decision through an inventory model. The model predicts that holding inventories is an optimal response to demand fluctuations and the presence of fixed production costs. Results also reveal a negative correlation between production and price decisions due to the substitutability relationship between those variables in the expected sales equation and in the marginal valuation of an additional unit of inventories.

We emphasize in the optimal responses of firms to cost-push stresses and demand contraction. We find that the dynamics of inventory distribution are particularly informative about the underlying transmission channel. To develop an identification strategy, we exploit not only difference in the aggregate behavior of variables but also the differences between responses of cross-section distribution variability and right tail. Then, we explore through which transmission channel the last financial crisis affected trade and manufacturing sectors of Peru and US. Results suggest that the main transmission channel of the crisis to Peruvian trade sector is a fixed cost increase because of the large inventory holdings increase, which is aligned with step rise of financial expenses to sales ratio during 2008. Meanwhile, Peruvian manufacturing sector (more clearly, the textile sector) seems to have been affected by a negative demand shock due to the reduction in inventories and the stickiness of mark-ups, this corresponds to the sharp contraction of this sector exports. In the case

of U.S. sectors both seem to experienced a cost-push stressss, because of the short term dynamics of mark-up response. This might be operating through the working capital channel.

Further researcher should include exploring the implications of the presented model in a general equilibrium framework, as well as calibration or estimation of the model (for which a more complete database is needed).

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